Technical Notes

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Thermophoretic Deposition in Crossflow over a Cylinder

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Introduction

THERMOPHORESIS causes small particles to be driven away from a hot surface and toward a cold one. This phenomenon has many practical applications. It affects the removal of small particles from gas streams, determines exhaust gas particle trajectories from combustion devices, and helps in studying the particulate material deposition on turbine blades. It is also of importance in the manufacture of fumed silica, carbon black, and titania particles for the paint industry.

All studies of thermophoretic deposition in external flow (except Homsy et al.[1] and Alam and Mehrotra[2]) are either for a zero pressure gradient in the boundary layer or for cases for which similarity solution is possible. Practical applications of thermophoresis in external flow, however, involve a nonzero pressure gradient for which no similarity solution is possible. Therefore, we study the thermophoretic deposition of aerosol particles in crossflow over a circular cylinder. Both theoretical and experimental pressure distributions in the hydrodynamic boundary layer over the cylinder are considered, unlike those in Homsy et al.[1] and Alam and Mehrotra,[2] where only the former is studied. A finite-difference method is used for the solution. The working fluid is taken to be air.

Governing Equations

Consider a gas containing suspended aerosol particles exposed to a circular cylinder of radius R in crossflow. The particle concentration is assumed to be diluted enough for the velocity and temperature profiles to pertain to those for a particle-free gas. For particles of unit density and 1 μm radius, this assumption limits the analysis to aerosol concentrations less than about 10^7 particles per cubic centimeter of the gas at normal temperature and pressure.[3] This is indeed the case for most thermophoretic applications. The velocity and temperature distributions in the laminar flow region over the cylinder are therefore governed by the usual boundary-layer equations.[4] The pressure gradient to be used in the momentum equation is found either from the potential flow solution over the cylinder or from the best fit for the experimental data.[4]

The temperature gradient established in the thermal boundary layer drives the particles either toward or away from the cylinder surface. The velocity acquired by the small particles relative to the gas velocity is known as the thermophoretic velocity v_t. Following the standard assumptions,[5] the conservation law for particle concentration, with the help of the continuity equation, reduces to

$$U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} + \frac{\partial}{\partial Y}(CV_T) = 0$$

where

$$X = x/R, \qquad Y = Re^{\frac{1}{2}}y/R, \qquad T = t/t_\infty,$$

$$U = u/u_\infty, \qquad V = Re^{\frac{1}{2}}v/u_\infty, \qquad Re = u_\infty R/\nu,$$

$$V_T = Re^{\frac{1}{2}}v_t/u_\infty = -(K/T)\,\partial T/\partial Y, \qquad C = c/c_\infty,$$

and where x is the streamwise coordinate measured from the stagnation point along the cylinder surface, y is the transverse coordinate, u_∞, t_∞, and c_∞ are the freestream velocity, temperature, and concentration, respectively, ν is the kinematic viscosity, and K is the thermophoretic coefficient. The boundary conditions are

$$C(0,Y) = 1 \qquad \text{and} \qquad C(X,\infty) = 1$$

For a cold wall a nonzero particle concentration at $Y = 0$ is determined by solving the preceding equations. Strictly speaking, this concentration is the one at the outer edge of the very thin Brownian sublayer, which is considered of negligible thickness in our analysis. For a hot wall the particle concentration at $Y = 0$ tends to $+\infty$ for $PrK > 1$ and to $-\infty$ for $PrK < 1$ (Goren[3]). A finite-difference method *was* used for the solution. Although details are available elsewhere,[5] it may be pointed out that a self-adaptive grid[6] was used in the Y direction. For accurate results, this is essential especially for the hot wall condition in which the particle concentration approaches $\pm\infty$ as the wall is approached.

Results and Discussion

The hydrodynamic and thermal boundary layers over a circular cylinder in crossflow have been investigated extensively by various researchers[4]; these results are therefore not presented here. It was found that the boundary layer separates over the cylinder at 105.5 and 81 deg from the stagnation point for theoretical and experimental pressure distributions, respectively. This is in perfect agreement with the values reported in the literature.[4]

Thermophoretic Velocity

Figure 1 shows the variation of thermophoretic velocity across the boundary layer at different locations over the cylinder. The curves below the horizontal axis correspond to cold wall ($T_w = 0.25$), and those above correspond to hot wall

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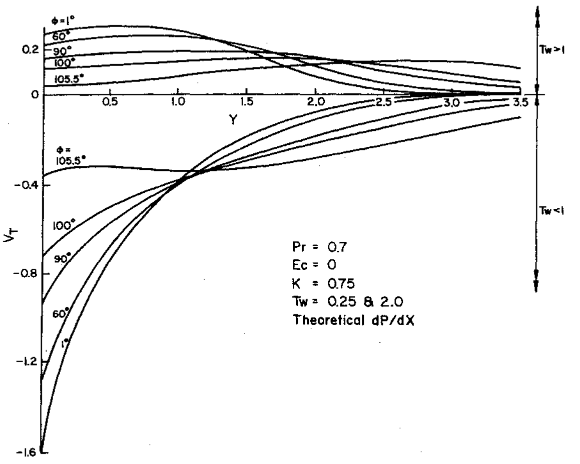

Fig. 1 Thermophoretic velocity distribution at various locations on a hot or cold cylinder.

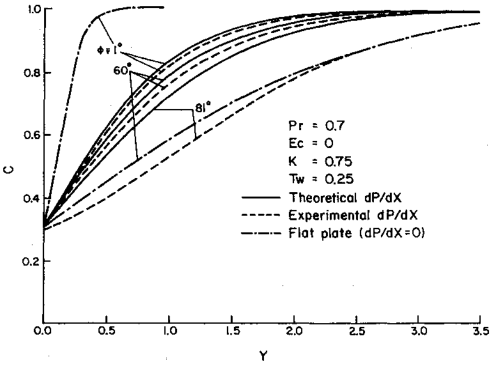

Fig. 2 Effect of pressure distribution on the particle concentration profiles over a cold cylinder.

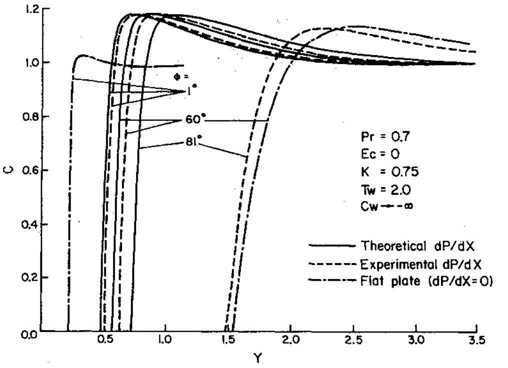

Fig. 3 Effect of pressure distribution on the particle concentration profiles over a hot cylinder.

($T_w = 2.0$) conditions. For cold wall conditions, the particle velocity vectors (U, $V + V_T$) are directed toward the cylinder suface. This is more so in the vicinity of the stagnation point. This causes the local deposition flux to decrease monotonically from a maximum value at the stagnation point. For hot wall conditions, the particle velocity vectors are directed away from the cylinder, causing a particle-free zone close to the cylinder surface (see Fig. 3).

Particle Concentration Profiles

For $T_w = 0.25$, $K = 0.75$, $Pr = 0.7$, and $Ec = 0$, Fig. 2 shows the dimensionless particle concentration profiles at various locations on the cold cylinder for both pressure distributions. To highlight the effect of $dP/dX = 0$, results are also shown for flow over a flat plate ($dP/dX = 0$). The presence of the stagnation point in the case of flow past a cylinder drastically affects the concentration curves (as well as velocity and temperature profiles) even far from the stagnation point. Note that $\phi = 1$ corresponds to $X = 1$. As explained earlier, a nonzero wall concentration, C_w, occurs in this case, but its value is almost independent of the location on the cylinder as well as of the pressure distribution. This is due to the coupling[7] between the temperature and concentration gradients at the cylinder surface. The simple relation given by Epstein et al.[7] relating C_w, Pr, K, and T_w for natural convection flow represents the present case as well within a relative error less than 8%, including the point of separation over the cylinder. It is found that C_w increases with T_w almost linearly as long as $T_w < 1$.

For a heated cylinder ($T_w > 1$) results obtained with $T_w = 2.0$, $K = 0.75$, $Pr = 0.7$, and $Ec = 0$ are illustrated in Fig. 3 for the two pressure distributions. The presence of a particle-free layer adjacent to the cylinder surface and concentration just outside this layer exceeding that in the freestream are characteristics of these results. The particle-free layer thickness increases with ϕ. However, the maximum particle concentration attained adjacent to this layer is almost independent of the location and pressure distribution. Results for other values of T_w are similar.

Conclusions

Thermophoretic deposition of aerosol particles in crossflow over a circular cylinder is determined for both the theoretical and experimental pressure distributions. A nonzero particle concentration is found to occur over a cold cylinder ($T_w < 1$), and its value is almost independent of the location on the cylinder. However, it increases almost linearly with T_w as long as $T_w < 1$. For a heated cylinder, a particle-free layer covers the cylinder surface, and this layer thickens rapidly near the point of separation.

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